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Radiative stabilization of warped space

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Abstract: Higher-dimensional field theory has been applied to explore various issues in recent particle physics such as the gauge hierarchy problem. In order for such approaches to be viable, a crucial ingredient is to fix the sizes of extra dimensions at some finite values, which sizes are generically free parameters in the theory. In this paper, we present several schemes to determine the radius of extra dimension in warped five-dimensional theory. In every case, a non-vanishing Fayet-Iliopoulos term for abelian gauge factor plays a crucial role for the radius stabilization. It is radiatively generated in the presence of charged matter fields and the compactification is therefore spontaneous, not forced by selected operators. The low-energy supersymmetry is broken or unbroken, and the radius can be fixed to give a small or large scale hierarchy without any fine tuning of parameters. We also discuss a model of the radius stabilization correlated with Yukawa hierarchy and supersymmetry breaking.

Keywords: Field Theories in Higher Dimensions, Supersymmetric Standard Model.

Contents

1. Introduction

Field theory in higher dimensions has been providing novel approaches to theoretical and phenomenological problems in recent particle physics. The existence of extra spatial dimensions beyond our fours is applied to various issues such as the generation of large scale hierarchies [1, 2]. In these approaches, a key ingredient is to determine the size of extra space so that they are viable approaches and do not conflict with current observations. For example, the Planck/weak mass hierarchy is attained by assuming that the radii of compactified dimensions are huge [1] or small but a bit larger than the Planck length [2]. The compactification radius is also conjectured to have anticipated values in other phenomenological discussions such as small neutrino masses [3], Yukawa hierarchies of quarks and leptons [4], and supersymmetry breaking [5]. Therefore adjusting the sizes of extra dimensions to desired values is one of the most important issues in higher-dimensional framework. There have been in the literature various resolutions to this stabilization problem in large- and small-sized extra dimensions [6].

In this paper, we present three different schemes to stabilize the radius modulus in five-dimensional supersymmetric theory with or without charged matter fields. One is based on the model with only boundary charged fields and another with only bulk fields. In every scheme, the Fayet-Iliopoulos (FI) term [7] in five-dimensional theory is found to play a crucial role for the radius stabilization. The resultant metric factor can be significant or nearly flat, depending on model parameters. It is stressed that the FI term is not introduced by hand in order to stabilize the radius. A non-vanishing FI term is radiatively generated even if it is set to zero in the classical lagrangian [8]. This is unlike the four-dimensional theory where a FI term does not receive any renormalization if theory has a vanishing gravitational anomaly [9]. The induced FI term depends on how charged matter multiplets are distributed in the extra dimensions and is therefore controllable. Further it is known in four-dimensional models that the FI term is connected to Yukawa hierarchy and supersymmetry breaking [10]. We construct a five-dimensional model for generating fermion mass hierarchy which is correlated with the radius stabilization. The model also predicts characteristic spectrum of sfermions and gauginos.

This paper is organized as follows. In section 2, we give a generic form of globally supersymmetric lagrangian for five-dimensional U(1) theory with the FI term. The simplest stabilization scheme is found in section 3 without introducing any matter fields. In section 4, a different scheme is presented to fix the size of extra dimension. The model contains bulk hypermultiplets with non-trivial wavefunctions whose forms are determined by FI-term coefficients. The bulk multiplets fix the distance between the two boundaries in terms of boundary couplings. In the vacuum of this model, supersymmetry is unbroken. With the generic lagrangian at hand, we show in section 5 that the radius can be stabilized by only boundary matter fields. That is established by analyzing the vacuum energy in effective four-dimensional theory with broken supersymmetry. We also construct a toy model for Yukawa hierarchy and supersymmetry breaking, deeply correlated with the radius stabilization phenomenon. Such a model predicts a new type of sparticle spectrum in low-energy effective theory. Section 6 is devoted to summarizing our results.

2. Five-dimensional U(1) gauge theory

We consider the globally supersymmetric abelian gauge theory in five dimensions. The fifth dimension is compactified on a line segment S^1/Z_2, where the radius of the circle S^1 is R. The radius R is a free parameter of the theory and corresponds to a massless moduli field T in four-dimensional effective theory ($R \equiv \mathrm{Re}\, T$). The fifth dimension y has two boundaries at $y = 0$ and πR. They are fixed points under the Z_2 orbifolding of physical spacetime. We are now interested in the case that the extra dimension has curved geometry. A particularly interesting example is the warped (AdS) geometry [2] whose line element is given by

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \tag{2.1}$$

where k is the AdS curvature and $\eta_{\mu\nu}$ the Minkowski metric in four dimensions. This background metric has been intensively studied for realistic model construction with the Planck/weak scale difference, quarks and leptons mass hierarchy, the cosmological constant problem, etc. In these approaches the radius R of the compact extra dimension was often assumed to have a desired value, and the radius stabilization is therefore one of the most important problems in constructing realistic 'brane-world' models. In this paper we present the schemes to stabilize R at a finite value due to the existence of U(1) gauge factor in supersymmetric warped dimensions.

We adopt the superspace formalism of higher-dimensional supersymmetry [11]. There are two types of supermultiplets generally introduced in five-dimensional theory; vector and hyper multiplets. A vector multiplet contains an $N = 1$ vector multiplet V and a chiral multiplet χ, whose auxiliary components are denoted by D and F_χ, respectively. A hypermultiplet consists of oppositely-charged two chiral multiplets ϕ and ϕ^c. In the superspace language, the most generic lagrangian for five-dimensional U(1) gauge theory is given by

$$L = L_V + L_H + L_{\rm UV}\delta(y) + L_{\rm IR}\delta(y-\pi R) + L_D, \tag{2.2}$$

$$L_V = \int d^2\theta \, \frac{1}{4g^2} W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \, \frac{e^{-2k|y|}}{g^2} \Big[\partial_y V - \frac{1}{\sqrt{2}}(\chi + \chi^\dagger)\Big]^2, \tag{2.3}$$

$$\begin{aligned} L_H = & \int d^4\theta \, e^{-2k|y|} (\phi^\dagger e^{qV}\phi + \phi^c e^{-qV}\phi^{c\dagger}) + \\ & + \int d^2\theta \, e^{-3k|y|} \phi^c \Big[\partial_y + \frac{q}{\sqrt{2}}\chi - \Big(\frac{3}{2} - c\Big) k\epsilon(y)\Big]\phi + \text{h.c.}, \end{aligned} \tag{2.4}$$

where g denotes the gauge coupling constant, and q and c are the U(1) charge and the bulk mass parameter of the chiral multiplet ϕ, respectively. The sign function $\epsilon(y)$ is inserted in order for the orbifold Z_2 invariance. We have also included the lagrangian for chiral multiplets confined on the UV ($y = 0$) and IR ($y = \pi R$) boundaries;

$$L_{\rm UV} = \int d^4\theta \, \phi_{\rm UV}^\dagger e^{q_{\rm UV}V} \phi_{\rm UV} + \int d^2\theta \, W_{\rm UV}(\phi, \phi_{\rm UV}) + \text{h.c.}, \tag{2.5}$$

$$L_{\rm IR} = \int d^4\theta \, e^{-2k\pi R} \phi_{\rm IR}^\dagger e^{q_{\rm IR}V} \phi_{\rm IR} + \int d^2\theta \, e^{-3k\pi R} W_{\rm IR}(\phi, \phi_{\rm IR}) + \text{h.c.}. \tag{2.6}$$

The orbifold boundary conditions are imposed on each supermultiplet. The vector multiplet V has the Neumann boundary conditions at both UV and IR branes and its superpartner multiplet χ has the Dirichlet ones because it contains the fifth component of the bulk gauge field. The boundary conditions of ϕ must be opposite to those of superpartner ϕ^c for respecting the Z_2 symmetry. The Z_2 boundary conditions break a half of bulk supersymmetry and thus the boundary lagrangians $L_{\rm UV}$ and $L_{\rm IR}$ preserve only the four-dimensional $N = 1$ supersymmetry. For example, Yukawa couplings of quarks and leptons are expected to come from these boundary interactions in the present framework. The boundary chiral multiplets $\phi_{\rm UV}$ and $\phi_{\rm IR}$ couple only to bulk multiplets with Neumann boundary conditions. We have assumed, for simplicity, that there are no y-derivative couplings of Z_2-odd chiral multiplets and no four-dimensional gauge fields on the boundaries, while these assumptions are irrelevant to the following discussion. The exponential warp factors are explicitly included in the above lagrangian. These warp factors describe the metric dependences, such as from $\sqrt{-\det g_{\mu\nu}}$, of the lowest component of each supermultiplet in the warped background. For other component fields, the proper metric factors in the warped five-dimensional action are obtained after some rescaling, for example, $D \to e^{-2k|y|}D$, $F_\phi \to e^{-k|y|}F_\phi$ for the auxiliary fields.

Since we now consider the abelian gauge theory, a FI term of vector multiplet V is gauge invariant in globally supersymmetric theory and can also be added to the lagrangian as

$$L_D = \int d^4\theta \, 2\xi V \, . \tag{2.7}$$

We have defined the coefficient ξ into which the metric warp factor is absorbed. Even if there is no FI term in classical lagrangian, it is radiatively generated via tadpole graphs of the D component where charged matter fields circulate in the loop. In the case of flat extra dimensions, its form was investigated [8, 12, 13] and found to reside only on the orbifold fixed points. Moreover the FI term vanishes in anomaly-free low-energy effective theory when one integrates out the fifth-dimensional physics. This is consistent with the fact that, in four-dimensional theory, a coefficient of radiatively-generated FI term is proportional to the sum of matter U(1) charges which also gives the mixed U(1)-gravitational anomaly. In five-dimensional theory on curved backgrounds including the warped geometry, the situation is rather different. Since the fifth direction is curved, the fundamental length depends on the position y. The implication of this fact appears through the metric-factor dependences in the FI-term calculation. In the warped geometry (2.1), the brane-localized FI terms are written as

$$\xi = \xi_{\rm UV}\delta(y) - \xi_{\rm IR}e^{-2k\pi R}\delta(y-\pi R) \, , \tag{2.8}$$

with the constant coefficients $\xi_{\rm UV}$ and $\xi_{\rm IR}$. If the FI term is set to vanish at classical level, radiative corrections give rise to $\xi_{\rm UV}$ and $\xi_{\rm IR}$ which are given by specific combinations of U(1) charges of bulk and boundary fields [14]. For a U(1) factor free from gravitational anomaly in low-energy theory, the two coefficients are equal to each other; $\xi_{\rm UV} = \xi_{\rm IR}$. The exponential factor in the second term of (2.8) indicates that the fundamental length is redshifted at the $y = \pi R$ boundary. This factor may be described by proper regularization, for example, à la Pauli-Villars, and more simply implemented by a position-dependent cutoff for the four-dimensional momentum in the one-loop calculations, which dependence is suggested by the AdS/CFT correspondence [15].[1] That has been recently confirmed by detailed analysis of five-dimensional supergravity [17]. In four-dimensional effective theory, the FI term does not vanish as the zero modes of vector multiplet have flat wavefunctions. As a result, either U(1) gauge symmetry or four-dimensional supersymmetry is broken at the scale of ξ. This reflects the known fact in four-dimensional theory that a FI term for anomaly-free U(1) gauge theory does not coexist with unbroken supersymmetry. That is, in four-dimensional supergravity theory, a FI term can be introduced only when U(1) is R symmetry or non-linearly realized, i.e. the U(1) gauge boson becomes massive. An important point here is that even if low-energy theory is totally free of anomalies like QED, the effective FI term is non-vanishing due to the curved extra dimension ($k \neq 0$) and has important phenomenological implications [14]. In this paper, we show that the presence of FI term also provides the stabilization mechanisms of the radius modulus field.

[1] A position-dependent value of FI term can also be seen from the theory-space approach to five-dimensional curved backgrounds [16].

3. Stabilization without bulk/boundary fields

Let us first see the simplest case where we have no charged matter fields in the theory. Integrating out the fifth-dimensional physics, we find that the presence of the FI term (2.8) leads to the potential

$$V(R) \;=\; \frac{g^2}{4\pi R}\left(\xi_{\rm UV}-\xi_{\rm IR}e^{-2k\pi R}\right)^2 . \qquad (3.1)$$

This vacuum energy depends on the radius R. It is therefore determined so that the vacuum energy is minimized. We find from (3.1) a possibility that the radius is fixed to a finite value

$$kR \;=\; \frac{1}{2\pi}\ln\left(\frac{\xi_{\rm IR}}{\xi_{\rm UV}}\right). \qquad (3.2)$$

The vacuum energy vanishes at this point which is the potential minimum in globally supersymmetric theory. Thus four-dimensional supersymmetry is unbroken while the radius is stabilized. Note that the limit $R\to\infty$ also gives a vanishing vacuum energy, where the low-energy gauge theory becomes a free theory. However the potential barrier between the two minima can be as high as $\xi_{\rm UV}^2$ whose natural size is around the Planck scale. Therefore the vacuum (3.2) might be made stable within the present age of the universe. Moreover the parameter region far away from the origin could be lifted by supersymmetry breaking which we have not included here. It is noticed that the existence of the vacuum (3.2) calls a restriction on the FI-term coefficients; $\xi_{\rm IR}/\xi_{\rm UV}>1$. For example, in case that low-energy theory is anomaly free, we have $\xi_{\rm UV}=\xi_{\rm IR}$ and hence the radius is not settled at a finite value. For radiatively-generated FI terms, the inequality $\xi_{\rm IR}\neq\xi_{\rm UV}$ is realized with 'anomalous' matter content. Then one should assume some anomaly cancellation mechanism that does not affect the FI terms. In this paper, we do not pursue such a possibility further. Instead we will discuss the radius stabilization with bulk/boundary matter fields in the presence of 'non-anomalous' FI term: $\xi_{\rm UV}=\xi_{\rm IR}\equiv\xi_{\rm FI}$.

4. Stabilization with bulk fields

In this section we present a scheme for stabilizing the size of warped extra dimension which involves only bulk hypermultiplets. A hypermultiplet which has non-trivial profile of bulk wavefunction connects two localized FI terms, and determines the distance between the boundaries. In this way the radius is fixed by the equations of motion of bulk fields together with their boundary conditions on the branes.

We consider the five-dimensional U(1) gauge theory with non-vanishing boundary FI terms. Its lagrangian is given by (2.3) and (2.4) as well as boundary superpotentials $W_{\rm UV}$ and $W_{\rm IR}$ for bulk chiral multiplets with even Z_2 parity. We do not include any boundary supermultiplets on the branes. The five-dimensional scalar potential is generally given by

$$V_{\rm 5D} \;=\; \frac{1}{2g^2}D^2+\frac{e^{-2k\pi R}}{g^2}|F_\chi|^2+\sum_\phi e^{-2k\pi R}(|F_\phi|^2+|F_{\phi^c}|^2)\,. \qquad (4.1)$$

The auxiliary fields D and F's are expressed in terms of bulk scalars through their equations of motion (see below). Here we have simply assumed that the fifth component of the U(1) vector field does not have a nonzero expectation value.

4.1 Model

The model we present in this section contains two bulk hypermultiplets with the following U(1) charges and bulk masses:

$$
\begin{aligned}
(\phi,\, \phi^c) &: \mathrm{U}(1) \text{ charge of } \phi = +q\,, \qquad \text{bulk mass} = c_\phi\,, \\
(\varphi,\, \varphi^c) &: \mathrm{U}(1) \text{ charge of } \varphi = -q\,, \qquad \text{bulk mass} = c_\varphi\,.
\end{aligned}
$$

Notice that two hypermultiplets have opposite U(1) charges so that they can form a mixing mass term. This is however just a simplifying assumption. We will mention other choices of U(1) charges in the end of this section and show that the charge assignment of hypermultiplets is irrelevant to the radius stabilization mechanism. The boundary conditions on the branes, namely, the orbifold parities are taken to be positive for ϕ and φ (therefore, negative for ϕ^c and φ^c), which lead to the zero modes of ϕ and φ in low-energy effective theory.

4.2 Unperturbed vacuum

There exists the supersymmetric vacuum in the presence of FI term (2.8) when the bulk scalars take appropriate expectation values so that the flatness conditions are satisfied. As mentioned in the previous section, we consider a conceivable case that $\xi_{\rm UV} = \xi_{\rm IR} \equiv \xi_{\rm FI}$. Without loss of generality, we take $q\xi_{\rm FI} > 0$ and then find that among the bulk matter scalars only Z_2-even φ develops a vacuum expectation value. The D and F flatness conditions now reduce to

$$
0 = -\partial_y(e^{-2k|y|}\Sigma) + \frac{qg^2e^{-2k|y|}}{2}|\varphi|^2 - g^2\xi_{\rm FI}[\delta(y) - e^{-2k\pi R}\delta(y-\pi R)]\,, \tag{4.2}
$$

$$
0 = \left[\partial_y - \frac{q}{2}\Sigma - \left(\frac{3}{2} - c_\varphi\right)k\epsilon(y)\right]\varphi\,. \tag{4.3}
$$

The field Σ is the real part of the neutral scalar in the chiral multiplet χ. It seems difficult to write down the generic solutions of these vacuum equations but we can analytically solve them for a specific value $c_\varphi = \frac{-1}{2}$. In this case, the solutions Σ_0 and φ_0 are given by

$$
\Sigma_0 = \frac{4ka_1}{q}\epsilon(y)e^{2k|y|}\tan\left(a_1e^{2k|y|} + a_2\right)\,, \tag{4.4}
$$

$$
\varphi_0 = \frac{4ka_1}{qg}e^{2k|y|}\frac{1}{\cos\left(a_1e^{2k|y|} + a_2\right)}\,, \tag{4.5}
$$

where a_1 and a_2 are the integration constants. These constants are determined by the boundary FI terms through the D-term equation (4.2) as

$$
-\frac{8ka_1}{qg^2}\tan(a_1 + a_2) = \xi_{\rm FI}\,, \qquad -\frac{8ka_1}{qg^2}\tan\left(a_1e^{2k\pi R} + a_2\right) = \xi_{\rm FI}e^{-2k\pi R}\,. \tag{4.6}
$$

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The existence of non-vanishing FI terms therefore fixes the unique supersymmetric vacuum away from the origin of the field space. When the warp factor is significant ($kR \gg 1$) and a FI term is small ($\xi_{\rm FI} \ll k^2$), the explicit forms of the integration constants are approximately given by

$$a_1 \simeq \left(\frac{\pi}{2} - \frac{qg^2\xi_{\rm FI}}{4k\pi}\right) e^{-2k\pi R}, \qquad a_2 \simeq \frac{\pi}{2} + \frac{4k\pi}{qg^2\xi_{\rm FI}}\, e^{-2k\pi R}. \tag{4.7}$$

The wavefunctions Σ_0 and φ_0 do not have singularities between the two boundaries. In the following analysis, we adopt this value $c_\varphi = \frac{-1}{2}$ as an example. It is however stressed that one may expect similar effects of radius stabilization for other generic values of the bulk mass parameters. As we will show below, the only required is the existence of non-trivial unperturbed solutions.

4.3 Radius determination

We introduce the following gauge-invariant superpotential terms onto the boundaries;

$$W_{\rm UV} = m_0\phi\varphi, \qquad W_{\rm IR} = m_\pi\phi\varphi. \tag{4.8}$$

It is noted that a supersymmetric mass term between ϕ and φ is forbidden by bulk AdS$_5$ supersymmetry and is forced to be confined on the boundaries, at which bulk supersymmetry is broken to the four-dimensional one. On the other hand, the mass terms $\phi\varphi^c$ and $\phi^c\varphi$ are allowed to exist by the bulk supersymmetry, but in the present model, the U(1) gauge invariance makes it vanish. Therefore (4.8) is the most generic superpotential for the matter fields involved. As we will see, the above potential terms play important roles of acting as the sources of ϕ which, in turn, stabilizes the radius and of lifting the φ direction from the equation of motion of ϕ.

Let us first see whether supersymmetry is broken in this model by examining the four-dimensional scalar potential. The D-term contribution to the scalar potential is written in the usual form with a non-vanishing FI term. The F-term contribution comes from the superpotential obtained by reducing (4.8) to the four-dimensional zero-mode part. Expanding $\phi(x,y) = \phi_4(x)\phi_y(y)$ and $\varphi(x,y) = \varphi_4(x)\varphi_y(y)$, it is given by

$$W_{\rm 4D} = [m_0\phi_y(0)\varphi_y(0) - m_\pi\phi_y(\pi R)\varphi_y(\pi R)]\phi_4(x)\varphi_4(x) \equiv m_{\phi\varphi}\,\phi_4(x)\varphi_4(x)\,, \tag{4.9}$$

where ϕ_y and φ_y are the solutions of their equations of motion. If the effective mass parameter $m_{\phi\varphi}$ is nonzero, ϕ_4 and φ_4 are lifted and supersymmetry is broken because the D-term equation enforces specific values on these fields. However $m_{\phi\varphi}$ now depends on the radius R, which is generically not a frozen parameter. The radius R thus fixes itself so as to give a vanishing effective mass $m_{\phi\varphi}$, for which the vacuum energy is minimized and supersymmetry is restored. In other words, if the radius modulus T is included as a dynamical variable, the minimization of scalar potential with respect to ϕ_4, φ_4 and T leads to a vanishing effective mass $m_{\phi\varphi}$ (at least local, supersymmetric vacuum).

Since supersymmetry is unbroken, all the equations of motion in the five-dimensional theory are

$$\begin{aligned} 0 = D \;&=\; -\partial_y(e^{-2k|y|}\Sigma) - \frac{qg^2}{2}e^{-2k|y|}(|\phi|^2 - |\phi^c|^2) + \frac{qg^2}{2}e^{-2k|y|}(|\varphi|^2 - |\varphi^c|^2) - \\ &\quad - g^2\xi_{\rm FI}[\delta(y) - e^{-2k\pi R}\delta(y-\pi R)], \qquad (4.10)\\ 0 = F_\chi^\dagger \;&=\; -\frac{qg^2}{\sqrt{2}}e^{-k|y|}(\phi^c\phi - \varphi^c\varphi), \qquad (4.11)\\ 0 = F_\phi^\dagger \;&=\; e^{-k|y|}\Big[\partial_y - \frac{q}{2}\Sigma - \Big(\frac{3}{2}+c_\phi\Big)k\epsilon(y)\Big]\phi^c - \Big[m_0\delta(y) - e^{-k\pi R}m_\pi\delta(y-\pi R)\Big]\varphi, \qquad (4.12)\\ 0 = F_\varphi^\dagger \;&=\; e^{-k|y|}\Big[\partial_y + \frac{q}{2}\Sigma - k\epsilon(y)\Big]\varphi^c - \Big[m_0\delta(y) - e^{-k\pi R}m_\pi\delta(y-\pi R)\Big]\phi, \qquad (4.13)\\ 0 = F_{\phi^c}^\dagger \;&=\; -e^{-k|y|}\Big[\partial_y + \frac{q}{2}\Sigma - \Big(\frac{3}{2}-c_\phi\Big)k\epsilon(y)\Big]\phi\,, \qquad (4.14)\\ 0 = F_{\varphi^c}^\dagger \;&=\; -e^{-k|y|}\Big[\partial_y - \frac{q}{2}\Sigma - 2k\epsilon(y)\Big]\varphi\,. \qquad (4.15) \end{aligned}$$

The localized operators enforce the specific boundary conditions on the parity-odd functions Σ, ϕ^c and φ^c such that

$$\Sigma \;=\; \epsilon(y)f_\sigma(y)\,, \qquad \phi^c \;=\; \epsilon(y)f_\phi(y)\,, \qquad \varphi^c \;=\; \epsilon(y)f_\varphi(y)\,, \qquad (4.16)$$

with the even functions $f_\sigma(y)$, $f_\phi(y)$ and $f_\varphi(y)$ which satisfy the conditions

$$\begin{aligned} 2f_\sigma(0) \;&=\; -g^2\xi_{\rm FI}\,, \qquad 2f_\sigma(\pi R) = -g^2\xi_{\rm FI}\,, \qquad (4.17)\\ 2f_\phi(0) \;&=\; m_0\varphi(0)\,, \qquad 2f_\phi(\pi R) = m_\pi\varphi(\pi R)\,, \qquad (4.18)\\ 2f_\varphi(0) \;&=\; m_0\phi(0)\,, \qquad 2f_\varphi(\pi R) = m_\pi\phi(\pi R)\,. \qquad (4.19) \end{aligned}$$

The F-term equations (4.12) and (4.13) are simplified in the bulk as

$$\begin{aligned} 0 &= \Big[\partial_y - \frac{q}{2}\Sigma - \Big(\frac{3}{2}+c_\phi\Big)k\epsilon(y)\Big]f_\phi\,, \qquad (4.20)\\ 0 &= \Big[\partial_y + \frac{q}{2}\Sigma - k\epsilon(y)\Big]f_\varphi\,. \qquad (4.21) \end{aligned}$$

Since we have $q\xi_{\rm FI} > 0$ without loss of generality, the scalar field ϕ do not develop vacuum expectation values. This is also understood from the view of four-dimensional effective theory. In turn, the equations (4.11) and (4.13) together with (4.19) mean $\varphi^c = 0$. The independent equations of motion now reduce to (4.10), (4.15), and (4.20) with $\phi = \varphi^c = 0$.

The vacuum solutions are explicitly derived by solving these equations in perturbation of $m_0, m_\pi \ll 1$. The leading-order solutions are given by the unperturbed ones (4.4), (4.5), and $f_\phi = 0$. It is interesting to notice that the radius determination does not require a precise form of Σ, which follows from the D-term equation (4.10). The formal solutions to (4.15) and (4.20) are

$$\varphi \;=\; A_\varphi\, e^{2k|y|}\exp\Big(\int^y \frac{q}{2}\Sigma\Big)\,, \qquad f_\phi \;=\; A_\phi\, e^{(\frac{3}{2}+c_\phi)k|y|}\exp\Big(\int^y \frac{q}{2}\Sigma\Big)\,, \qquad (4.22)$$

with A_φ and A_ϕ being the integration constants. Inserting the solutions into (4.18), we find that the boundary conditions of ϕ^c determine the value of R:

$$kR = \frac{\ln\left(\frac{m_\pi}{m_0}\right)}{(c_\phi - \frac{1}{2})\pi}. \qquad (4.23)$$

The boundary conditions also constrain the ratio of integration constants as $A_\phi/A_\varphi = O(m_0) \ll 1$. Their individual values are fixed by the D-term equation which is satisfied by $O(m_{0,\pi}^2)$ fluctuation of Σ around the unperturbed solution (4.4). The fact that $A_\phi \ll 1$ ensures the relevance of the perturbative analysis. The above derivation makes it clear that explicit solutions to the equations of motion are not needed to find a stabilized value of the radius. In fact, the radius in the minimum can easily be evaluated for a generic value of c_φ. That is, a similar analysis shows that R is determined so that $kR = \ln(\frac{m_\pi}{m_0})/(c_\phi + c_\varphi)\pi$.

Several comments are in order. At least at this order of perturbation, the stabilized value of the radius does not seem to depend on the FI term. It is however noticed that the wavefunction factors A's depend on the FI term via a_1. If one turns off the FI term ($\xi_{\rm FI} \to 0$), the vacuum goes to the origin of field space, that is, $a_1 \to 0$ (see (4.6)). Consequently, the expectation value of ϕ^c also vanishes which cannot lead to the radius determination (4.23). In this way the existence of non-vanishing FI term is crucial for the stabilization of radius modulus field. Secondly, as for the parameters m_0 and m_π, realizing a significant warp factor does not need any fine tuning. For example, the values $c_\phi \simeq 0.6$ and $\frac{m_\pi}{m_0} \simeq 20$ give $e^{k\pi R} \sim 10^{15}$ in (4.23). What is needed is a parameter choice of order $O(0.1)$, which is similar to that in the original Randall-Sundrum model where $kR \sim O(10)$ is assumed for solving the gauge hierarchy problem. Of course, a radius stabilization with no significant warp factor $e^{k\pi R} \sim O(1)$ is easier to be achieved. Finally, we comment on the possibility for other choices of U(1) charges. When the bulk mass c_ϕ is exactly one half, the equation (4.23) implies that the radius is not stabilized. This is however simply because of our U(1) charge assignment. If one supposes the U(1) charge of ϕ is $+nq$ for example, the gauge-invariant boundary superpotentials take the form $W \sim \phi\varphi^n$. In this case (also with a general unfixed c_φ), the stabilized value of R is replaced with

$$kR = \frac{\ln\left(\frac{m_\pi}{m_0}\right)}{(c_\phi + c_\varphi + 2 - 2n)\pi}. \qquad (4.24)$$

Thus the radius is still stabilized as long as there is no principle relating bulk mass parameters and U(1) charges in a specific way.

5. Stabilization with boundary fields

In this section we examine whether the radius modulus can be stabilized only with boundary field dynamics unlike the model presented in the previous section. In this case, the stabilization procedure is to look for the minimum of four-dimensional effective potential of the radius modulus field. Let us first derive low-energy effective theory for generic boundary superpotential terms. We assume that supergravity effects except for the radius modulus are irrelevant to stabilization. Integrating out the fifth dimension, we obtain the low-energy effective theory of the U(1) multiplet zero modes, boundary matter multiplets, and the radius modulus T. The effective lagrangian is derived from (2.3), (2.5)–(2.8):

$$\begin{aligned}\mathcal{L}_{4\mathrm{D}} &= \int d^2\theta\, \frac{\pi T}{2g^2} W^\alpha W_\alpha + \text{h.c.} + \int d^4\theta \left[\phi_{\mathrm{UV}}^\dagger e^{q_{\mathrm{UV}}V}\phi_{\mathrm{UV}} + e^{-k\pi(T+T^\dagger)}\phi_{\mathrm{IR}}^\dagger e^{q_{\mathrm{IR}}V}\phi_{\mathrm{IR}}\right] + \\ &\quad + \int d^2\theta \left[W_{\mathrm{UV}}(\phi_{\mathrm{UV}}) + e^{-3k\pi T}W_{\mathrm{IR}}(\phi_{\mathrm{IR}})\right] + \text{h.c.} + \\ &\quad + \int d^4\theta \Big(2\xi_{\mathrm{FI}}V - \frac{6M^3}{k}\Big)\left[1 - e^{-k\pi(T+T^\dagger)}\right], \end{aligned} \tag{5.1}$$

where M is the fundamental scale in five-dimensional theory. We have included the proper Kähler term of the radius modulus field in the warped background [18]. It is assumed that extra bulk dynamics to lead to potential terms of T is not introduced. As mentioned in the previous section, however, the FI term is automatically generated in the presence of charged matter fields and provides T-dependent terms. The scalar potential is obtained by integrating out all the auxiliary components

$$\begin{aligned} V_{4\mathrm{D}} &= \frac{g^2}{4\pi R}\left[\frac{q_{\mathrm{UV}}}{2}|\phi_{\mathrm{UV}}|^2 + \frac{q_{\mathrm{IR}}}{2}e^{-2k\pi R}|\phi_{\mathrm{IR}}|^2 + \xi_{\mathrm{FI}}(1-e^{-2k\pi R})\right]^2 + \\ &\quad + \left|\frac{\partial W_{\mathrm{UV}}}{\partial\phi_{\mathrm{UV}}}\right|^2 + e^{-4k\pi R}\left|\frac{\partial W_{\mathrm{IR}}}{\partial\phi_{\mathrm{IR}}}\right|^2 + \frac{ke^{-4k\pi R}}{6M^3}\left|3W_{\mathrm{IR}} - \phi_{\mathrm{IR}}\frac{\partial W_{\mathrm{IR}}}{\partial\phi_{\mathrm{IR}}}\right|^2 . \end{aligned} \tag{5.2}$$

For later discussion, we present the equations of motion for the auxiliary fields;

$$D = \frac{-g^2\xi_{\mathrm{FI}}}{2\pi R}(1-e^{-2k\pi R}) - \frac{q_{\mathrm{UV}}g^2}{4\pi R}|\phi_{\mathrm{UV}}|^2 - \frac{q_{\mathrm{IR}}g^2}{4\pi R}e^{-2k\pi R}|\phi_{\mathrm{IR}}|^2 , \tag{5.3}$$

$$F_{\phi_{\mathrm{UV}}}^\dagger = -\frac{\partial W_{\mathrm{UV}}}{\partial\phi_{\mathrm{UV}}} , \tag{5.4}$$

$$F_{\phi_{\mathrm{IR}}}^\dagger = -e^{-k\pi R}\frac{\partial W_{\mathrm{IR}}}{\partial\phi_{\mathrm{IR}}} + \frac{ke^{-k\pi R}}{6M^3}\phi_{\mathrm{IR}}^\dagger\Big(3W_{\mathrm{IR}} - \phi_{\mathrm{IR}}\frac{\partial W_{\mathrm{IR}}}{\partial\phi_{\mathrm{IR}}}\Big) , \tag{5.5}$$

$$F_T^\dagger = \frac{e^{-k\pi R}}{6\pi M^3}\Big(3W_{\mathrm{IR}} - \phi_{\mathrm{IR}}\frac{\partial W_{\mathrm{IR}}}{\partial\phi_{\mathrm{IR}}}\Big) , \tag{5.6}$$

where we have simply assumed that the graviphoton field does not have a nonzero expectation value. The scalar potential is a function of boundary scalar fields and the modulus R. Assuming an appropriate form of boundary superpotentials, we first minimize the potential (5.2) with respect to matter scalars and find the minimum value $V(R)$ of the potential, which generally depends on R. Then doing the minimization of $V(R)$, we obtain the vacuum with a stabilized value of R.

From the generic form of the scalar potential (5.2), we have several observations for the radius stabilization: (i) First, unless the FI term is present, the radius is not stabilized. For a vanishing value of ξ_{FI}, there is a D-flat direction in the potential. Along this direction, the potential has the R dependences only in the form of $e^{-4k\pi R}$ and the vacuum goes to infinity. (ii) Another observation is that, unless supersymmetry is broken, the radius is not stabilized at a finite value. Generally speaking, the scalar potential vanishes for unbroken supersymmetry and hence cannot fix the radius. In the present case, the FI term itself gives rise to a non-vanishing potential. However it is a monotonous function of R and consequently, the radius R is fated to have a runaway behavior.

We thus find that, for stabilizing the radius modulus, the FI term must be present and also four-dimensional supersymmetry must be broken (leading to non-vanishing vacuum energy). An interesting point is that, in curved five-dimensional theory, a FI term is automatically induced and can cause required supersymmetry breaking. Note that the above arguments are applied to the models with boundary multiplets only, and therefore including hypermultiplets and/or bulk dynamics may change the conclusion. For example, as in the model of section 4, non-trivial R-dependences of bulk-field wavefunctions generate R-dependent (super)potential terms, which can lead to (supersymmetric) radius stabilization. We have shown in the above that this cannot be obtained by boundary matter only and supersymmetry needs to be broken.

5.1 Radius determination

As we mentioned, four-dimensional supersymmetry must be broken to stabilize the size of the compact fifth dimension. In the absence of charged matter fields, the FI term leads to a non-vanishing potential for the radius modulus, as discussed in section 3. However it does not have any minimum with respect to R for anomaly-free theory. Moreover it might be unfavorable with such a vacuum energy that supersymmetry is broken at a high-energy scale. In this section, we are thus interested in including the charged matter contribution. With a non-vanishing FI term, the D-flatness condition points to the unique vacuum with nonzero expectation values of charged matter scalars. Therefore adding appropriate perturbation to superpotential terms, the charged field directions are lifted and hence supersymmetry is broken, as needed.

The most simple case is to introduce on the $y = 0$ boundary a vector-like chiral multiplets $\phi_{\rm UV}$ and $\bar{\phi}_{\rm UV}$ (with U(1) charges $+q_{\rm UV}$ and $-q_{\rm UV}$ respectively) and their mass term:[2]

$$W_{\rm UV} = m\phi_{\rm UV}\bar{\phi}_{\rm UV}\,. \tag{5.7}$$

Together with the D-term potential, one can see that supersymmetry is broken by small perturbation ($m \sim$ TeV). It is however found from (5.2) that the modulus potential in this case has the maximum only. We thus incorporate a constant superpotential on the $y = \pi R$ boundary: $W_{\rm IR} = \omega$. Such a constant superpotential can be obtained in various ways and here we do not consider any details of its origin. Without loss of generality, $q_{\rm UV}\xi_{\rm FI}$ is taken to be positive and the vacuum is given by

$$\phi_{\rm UV} = 0, \qquad |\bar{\phi}_{\rm UV}|^2 = \frac{2\xi_{\rm FI}}{q_{\rm UV}}(1 - e^{-2k\pi R}) - \frac{8\pi R m^2}{q_{\rm UV}^2 g^2}\,. \tag{5.8}$$

[2]Other examples are to introduce a vector-like chiral multiplets on the $y = \pi R$ boundary, to introduce only a constant superpotential, and so on. We find that, in either of these cases, the radius modulus is not stabilized at a finite value.

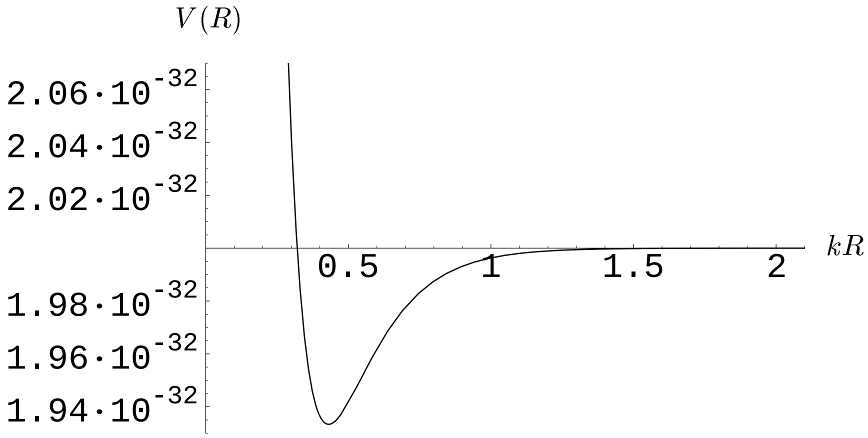

Figure 1: The four-dimensional vacuum energy as a function of the size of the fifth dimension (5.9). In the figure, we take $m = 10^{-15}$, $\omega = 10^{-16}$, $k = 0.1$, and $\xi = 0.01$ in the unit of the fundamental scale M.

The vacuum energy, which depends on the radius R, becomes

$$V(R) \;=\; \frac{2m^2}{q_{\rm UV}}\xi_{\rm FI}(1-e^{-2k\pi R}) - \frac{4\pi R m^4}{q_{\rm UV}^2 g^2} + \frac{3k|\omega|^2}{2M^3}e^{-4k\pi R}\,. \tag{5.9}$$

Minimizing the vacuum energy determines the value of R. We find the solutions to $\frac{\partial V(R)}{\partial R} = 0$;

$$e^{2k\pi R} \;=\; \frac{k\xi_{\rm FI}q_{\rm UV}g^2}{2m^2}\left(1 \pm \sqrt{1-\frac{6\omega^2}{g^2\xi_{\rm FI}^2 M^3}}\right). \tag{5.10}$$

In this vacuum with a stabilized R, supersymmetry is broken by nonzero D and F terms;

$$D = \frac{-2m^2}{q_{\rm UV}}, \qquad F_{\phi_{\rm UV}} \;\simeq\; -m\sqrt{\frac{\xi_{\rm FI}}{q_{\rm UV}}}, \qquad F_T = \frac{\omega^* e^{-k\pi R}}{2\pi M^3}\,. \tag{5.11}$$

There are two typical scales of the constant term ω such that F_T/R is on the order of supersymmetry-breaking scale O(TeV). The first case is given by a suppressed value of ω compared to the fundamental scale M^3. In this case, it is easily found by analyzing the second derivative of the potential that the minimum of $V(R)$ is given by $e^{2k\pi R} \simeq \frac{3q_{\rm UV}k|\omega|^2}{2m^2\xi_{\rm FI}M^3}$. Therefore the radius is stabilized around the value

$$kR \;\simeq\; O(1)\,, \tag{5.12}$$

and a large metric warp factor does not arise. The supersymmetry-breaking contribution from the radius modulus is found to be $F_T \sim \omega/M^3 \ll 1$. Figure 1 shows an explicit form of the vacuum energy (the potential of the radius modulus) for a suppressed value of ω. The

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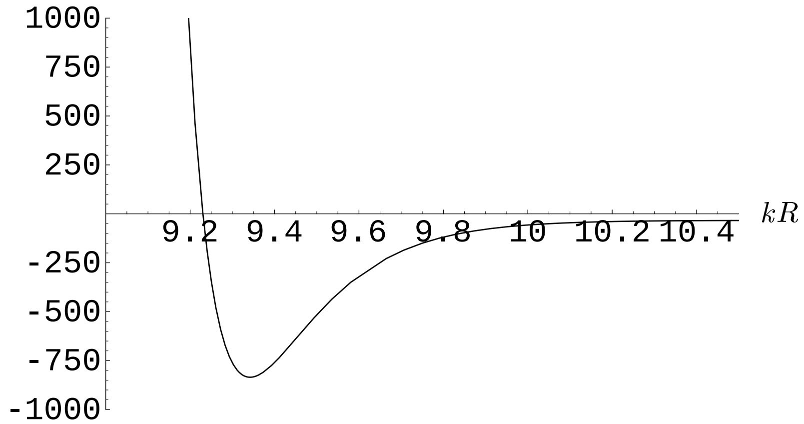

Figure 2: Typical behavior of the vacuum energy $V(R)$ for $m = 10^{-15}$, $\omega = 0.01$, $k = 0.1$, and $\xi = 1$ in the unit of M. (The figure plots the potential from which we have subtracted a radius-independent constant $\frac{m^2\xi_{\rm FI}}{q_{\rm UV}}$ and normalized it by $\frac{4m^4}{kq_{\rm UV}^2 g^2}$.) For a smaller value of $\omega/\xi_{\rm FI}$, the valley of the minimum becomes steeper.

stabilized modulus obtains the mass squared $m_T^2 = \frac{e^{2k\pi R}}{6k\pi^2 M^3}\frac{\partial^2 V(R)}{\partial R^2}$ which is always positive definite at the minimum. In the parameter region we now consider, it is approximately given by

$$m_T^2 \simeq \frac{4m^4\xi_{\rm FI}^2}{9q_{\rm UV}^2\omega^2} \simeq O\left((\text{TeV})^2\right), \tag{5.13}$$

with the canonical kinetic term of the radius modulus.

Another typical scale of ω is a natural scale in the theory, namely, $\omega \sim M^3$. In this case, the solution (5.10) means that the minimum is around $e^{2k\pi R} \simeq \frac{k\xi_{\rm FI}q_{\rm UV}g^2}{m^2}$ and therefore,

$$kR \simeq \frac{1}{2\pi}\ln\left(\frac{M}{\text{TeV}}\right)^2 \sim 10\,. \tag{5.14}$$

As a result, the metric warp factor gives significant effects, and also we have a suppressed value of radius modulus F term; $F_T \sim e^{-k\pi R} \sim (\text{TeV})/M$. In figure 2, we show a typical behavior of the vacuum energy $V(R)$. The mass of the radius modulus with the proper normalization in this region is

$$m_T^2 \simeq \frac{4m^4 e^{2k\pi R}}{3q_{\rm UV}^2 g^2 M^3} \simeq O\left((\text{TeV})^2\right). \tag{5.15}$$

Again we have a TeV-scale massive modulus field. For a large metric factor, KK-excited modes have suppressed masses above $O(\text{TeV})$ in four-dimensional theory. It might require a careful treatment to examine whether the effective lagrangian of zero modes is valid in this parameter region. In addition, the minimum might not be so steep that it is not meta-stable within the cosmological evolution. However a higher potential barrier can be achieved by a smaller value of the constant superpotential, and a unstable vacuum is easily avoided.

	$(\phi_L, \bar{\phi}_L)$	ϕ_R	H	ϕ	$\bar{\phi}$
U(1)	$(q_L, -q_L)$	q_R	0	1	-1

Table 1: The U(1) charge assignment. We take the charge of the Higgs field zero, for simplicity, and the matter charges q_L and q_R are positive.

5.2 Towards realistic models

We have shown that a nonzero FI term can stabilize the radius modulus to a realistic value. As well known in four-dimensional theory, the FI term is capable of explaining Yukawa hierarchy of quarks and leptons and also of providing interesting sparticle spectrum. In this subsection, we present a toy model towards constructing realistic theory in higher dimensions where a single existence of FI term has various important implications to phenomenology.

Let us consider one-generation 'lepton' multiplets ϕ_L, ϕ_R, and 'Higgs' H as well as a vector-like multiplets ϕ and $\bar{\phi}$. The latters play the radius stabilizer discussed in the previous section. For simplicity, we focus only on the U(1) factor and ignore the standard model gauge groups. Incorporating these gauge factors is rather straightforward. Now suppose that ϕ_L comes from a five-dimensional hypermultiplet $(\phi_L, \bar{\phi}_L)$ with orbifold parities $(+,-)$. So ϕ_L contains a massless mode in four-dimensional effective theory. All other multiplets ϕ_R, H, ϕ and $\bar{\phi}$ are assumed to be confined on the UV boundary. The U(1) charges of these multiplets are listed in the table 1. As in usual four-dimensional case, the U(1) charges of matter fields are taken as positive, which will be important to have non-vanishing Yukawa couplings, positive sfermion masses squared, and also the potential analysis in the previous section to be valid. The charge q has been set to $+1$ without loss of any generalities. Note that, with only these multiplets at hand, the zero-mode effective U(1) theory is anomalous. But some anomaly cancellation may easily be assumed, for example, introducing additional charged multiplets. An important point here is that, even if effective four-dimensional theory is anomaly free, an induced FI term can be nonzero due to the curved extra dimension. Note also that there are no gauge anomalies for the standard gauge groups, if included, provided that anomalies are cancelled within massless modes [19, 12, 14, 20]. Therefore in case that light-mode spectrum is that of the standard model, we do not worry about gauge (and gravitational) anomalies for any field configurations in the extra dimension.

The FI term may be radiatively generated even for anomaly-free particle contents. In the U(1) theory above, the one-loop contribution is given by (2.8) with $\xi_{\rm UV} = \frac{(q_L+2q_R)\Lambda^2}{32\pi^2}$ and $\xi_{\rm IR} = \frac{-q_L\Lambda^2}{32\pi^2}$ where Λ is near the fundamental scale M. As seen in the previous section, the successful radius stabilization needs a positive value of $\xi_{\rm IR}$, which is not satisfied in the present form. ($\xi_{\rm UV}$ must also be positive to have Yukawa couplings and supersymmetry breaking.) A simple way to cure this problem is to introduce charged hypermultiplets which have even (odd) orbifold parity at the UR (IR) boundary, or vice verse. These additional multiplets contribute to the FI coefficients $\Delta\xi_{\rm UV} = \Delta\xi_{\rm IR} = \frac{Q\Lambda^2}{32\pi^2}$ where Q is the sum of U(1) charges of added multiplets. Moreover they contain no zero modes and do not change the standard model spectrum. With this implementation, both FI-term coefficients can safely be positive. In the previous analysis, $\xi_{\rm FI}$ is replaced with $\xi_{\rm IR}$ in the solution (5.10) and the expressions of the radion mass, but in the expectation values of $\bar{\phi}_{\rm UV}$ and $F_{\phi_{\rm UV}}$, $\xi_{\rm FI}$ is approximately given by $\xi_{\rm UV}$, which is suitably positive if $\xi_{\rm IR}$ is made positive.

The Yukawa couplings for matter multiplets are described by gauge-invariant higher-dimensional operators [21] on the $y=0$ boundary:

$$W_{\rm UV} = h\Big(\frac{\bar{\phi}}{M}\Big)^{q_L+q_R}\phi_L\phi_R H\,, \tag{5.16}$$

where h is the $O(1)$ coupling constant. If one included other generations originated from bulk hypermultiplets, their Yukawa couplings are allowed by bulk supersymmetry only on the boundaries as (5.16). The potential analysis shows that only $\bar{\phi}$ develops a non-vanishing expectation value, i.e. (5.8).[3] As a result, the above operator gives an effective Yukawa coupling $y \sim h(\frac{\langle\bar{\phi}\rangle}{M})^{q_L+q_R}$. The expectation value of $|\bar{\phi}|^2$ is proportional to the FI term and thus an one-loop order quantity. Therefore we obtain $\frac{\langle\bar{\phi}\rangle}{M} \equiv \lambda \sim O(0.1)$ that is just suitable for describing realistic Yukawa hierarchies. Furthermore, in this model, there is an additional possibility to have Yukawa suppression unlike in pure four-dimensional theory. That is a wavefunction factor of bulk hypermultiplet zero modes. The zero-mode wavefunction ϕ_{L_0} depends on its bulk mass c and U(1) charge, and is given by [14]

$$\phi_{L_0} \;=\; N_0 \exp\left[\Big(\frac{1}{2}-c\Big)k|y| + q_L a e^{2k|y|}\right], \tag{5.17}$$

with N_0 being the normalization constant determined by $\int dy|\phi_{L_0}|^2 = 1$ and roughly given by $N_0^2 \simeq \frac{(1-2c)k}{e^{(1-2c)k\pi R}-1}$. The present setup predicts $a = \frac{g^2\xi_{\rm IR}}{8k}e^{-2k\pi R}$ derived from the background expectation value of Σ, which is fixed by the five-dimensional D-term equation. The second term in the bracket is thus tiny in all region of the fifth dimension and can be dropped. We then find that N_0 provides additional suppression of the effective Yukawa coupling in the present model. For $c > \frac{1}{2}$, the corresponding zero mode is localized at $y=0$ and yields no suppression of Yukawa couplings which come from the operator on the $y=0$ boundary. On the other hand, the $c < \frac{1}{2}$ case gives a Yukawa suppression by the factor $N_0 \sim e^{(\frac{1}{2}-c)k\pi R} \ll 1$. This reflects the fact that the zero mode is peaked at away from the $y=0$ boundary. In the conformal limit $c=\frac{1}{2}$, the normalization constant N_0 becomes a volume-suppression factor $\frac{1}{\sqrt{\pi R}}$ as in the case of flat extra dimension.

Supersymmetry-breaking spectrum is related to the radius stabilization and Yukawa coupling structure. According to (5.11), there are three types of contributions to supersymmetry-breaking parameters. Since we now introduced only boundary multiplets, the equation of motion of Σ in five dimensions implies that the auxiliary field D has a flat wavefunction in the fifth direction. Therefore the D-term contribution is universal to all charged scalars in the theory. The F component of ϕ provides soft masses and trilinear couplings of scalar fields from superpotential and/or Kähler terms. It is found that they are

[3] The scalar $\bar{\phi}_L$ also has a negative U(1) charge and might be worried to obtain a nonzero expectation value. However the equation of motion for the ϕ_L scalar implies that this is not the case as long as there is no source term of ϕ_L on the UV boundary.

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higher-dimensional operators and suppressed by powers of $\lambda = \frac{\langle\bar{\phi}\rangle}{M}$ compared to the leading D-term contribution. Ignoring these higher-dimensional corrections,[4] non-holomorphic scalar masses are given by

$$\begin{aligned} m_L^2 &= -q_L D + \partial_T\partial_{\bar{T}} \ln|N_0(T,\bar{T})|^2 |F_T|^2 \\ &\simeq 2q_L m^2 + \left| \frac{(c-\frac{1}{2})k\pi R}{\sinh\left[(c-\frac{1}{2})k\pi R\right]} \frac{F_T}{2R} \right|^2 \qquad \text{(bulk scalars)}\,, \qquad (5.18) \\ m_R^2 &= 2q_R m^2 \qquad (y=0 \text{ boundary scalars})\,, \qquad (5.19) \end{aligned}$$

where $N_0(T,\bar{T})$ is the appropriate superspace extension of the normalization constant N_0. The first terms are the D-term contributions which are positive definite ($q_{L,R} > 0$). The second term in the bulk scalar mass m_L^2 comes from the radius modulus F term (5.11). We have dropped the Σ contribution in m_L^2 since it is suppressed in the wavefunction factor ϕ_{L_0} as discussed above. The boundary scalars at $y = 0$ receive no F_T contribution as seen from the lagrangian (5.1) and have rather different spectrum than those of bulk scalar fields.

In case that quarks and leptons originate from bulk hypermultiplets, the standard model gauge multiplets must also reside in the five-dimensional bulk. Then the gauginos obtain supersymmetry-breaking masses from two contributions; the radius modulus F term and higher-dimensional operators $\int d^2\theta\, c_i \frac{\phi\bar{\phi}}{M^2} W^{\alpha i} W_\alpha^i$. The masses of zero-mode gauginos $M_{1/2}^i$ are given by

$$M_{1/2}^i = \frac{-c_i g^2 \lambda^2 m}{\pi R} + \frac{F_T}{2R}\,. \qquad (5.20)$$

Let us concentrate on the vacuum with $kR \sim O(1)$. In this vacuum, the radius modulus F term is

$$\frac{F_T}{R} \simeq \frac{\omega^* e^{-k\pi R}}{2\pi R M^3} \simeq \frac{\lambda m}{\sqrt{12\pi^2 MR}} \ll m\,. \qquad (5.21)$$

For bulk scalar masses, the dominant part therefore comes from the D-term contribution and the spectrum is similar to four-dimensional anomalous U(1) models. The D-term contributed scalar masses have rich phenomenological implications such as flavor violation [22]. On the other hand, the two contributions to gaugino masses in (5.20) are comparable in size or the F_T contribution can be dominant if low-energy effective theory has a weak gauge coupling constant $\frac{g^2}{\pi R} \ll 1$. The gauginos are found to have rather non-universal (non-unified) mass spectrum in this scenario.

As seen in this toy model, the existence of FI term provides various schemes to discuss phenomenological issues in higher-dimensional theory. It can stabilize the sizes of extra dimensions, create Yukawa hierarchies, and predict characteristic sparticle spectrum testified in future particle experiments. Therefore more realistic model construction along this line may deserve to be investigated.

[4]Bulk fields with vanishing U(1) charges, like scalar top quark, might receive the dominant soft masses from higher-dimensional Kähler terms involving ϕ.

6. Summary

In this work we have discussed the Fayet-Iliopoulos D term as a possible origin of radius stabilization in brane world models. We have presented three different schemes for the stabilization. The simplest case has no matter multiplets to stabilize the radius, but the theory needs some cancellation mechanism of gravitational amomaly. The one of the others contains bulk hypermultiplets, whose non-trivial wavefunctions connect the two boundaries and then fix the size of the extra dimension in terms of boundary couplings. On the other hand, the third model involves only boundary dynamics and does not need the presence of bulk matter fields. The radius is, in this case, determined so that the vacuum energy is minimized after supersymmetry breaking. Every scheme can lead to a significant warp factor or near-flat metric, depending on the model parameters.

It should be noted that, in any model, the FI term is not a device introduced just in order to stabilize the radius modulus. A non-vanishing FI term is radiatively generated even if it is set to be zero at classical level. An induced FI term depends on how charged matter multiplets are distributed in the extra dimensions and is therefore controllable. Moreover it is known in four-dimensional models that the FI term is deeply connected with Yukawa hierarchy and supersymmetry breaking. We have presented a toy model for fermion Yukawa hierarchy correlated to the radius stabilization. The model also predicts characteristic spectrum of sfermions and gauginos.

In the model we have drawn in section 4, the bulk scalar fields develop non-trivial wave-function profiles in the extra dimension and four-dimensional supersymmetry is unbroken. When included supersymmetry breaking, it might give impacts on sparticle spectroscopy and also deserve cosmological considerations. We leave these phenomenological analysis to future investigations.

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